29.69. Model: Assume the thin rod is a line of charge with uniform linear charge density.

Visualize: Please refer to Figure P29.69. The point P is a distance d from the origin. Divide the charged rod into N small segments, each of length Δx and with charge Δq. Segment i, located at position x_i, contributes a small amount of potential V_i at point P.

Solve: The contribution of the ith segment is

$$V_i = \frac{\Delta q}{4\pi\varepsilon_0 r_i} = \frac{\Delta q}{4\pi\varepsilon_0 (d - x_i)} = \frac{Q\Delta x/L}{4\pi\varepsilon_0 (d - x_i)}$$

Where $\Delta q = \lambda\Delta x$ and the linear charge density is $\lambda = Q/L$. We are placing the point P at a distance d rather than x from the origin to avoid confusion with x_i. The V_i are now summed and the sum is converted to an integral giving

$$V = \frac{Q}{4\pi\varepsilon_0 L}\int_{-L/2}^{L/2}\frac{dx}{d - x} = \frac{Q}{4\pi\varepsilon_0 L}\left[-\ln(d - x)\right]_{-L/2}^{L/2} = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{d + L/2}{d - L/2}\right)$$

Replacing d with x, the potential due to a line charge of length L at a distance x along the axis is

$$V = \frac{Q}{4\pi\varepsilon_0 L}\ln\left(\frac{x + L/2}{x - L/2}\right)$$